**Mathematical details of effect sizes used in this paper**

**x.1 Standardized effect size typology**

Over the last 30 years an increased focus has been placed on the reporting and interpretation of effect sizes as an important part of the development of a cumulative and interpretable research literature {Kruschke, 2017 #105;e.g.`, \Cumming, 2013 #158;Wilkinson, 1999 #566;Hedges, 1981 #786}. Effect sizes can be expressed in standardised or unstandardized units. Unstandardized effect sizes (e.g., mean differences) are presented in the units the measured variables, and may be particularly useful when the units of analysis are directly interpretable (e.g., income, IQ scores, measures of height or weight). Standardised effect sizes (e.g., Cohen’s *d* for mean differences) have several distinct uses. Some measures may be useful for direct interpretation when the units of measurement are not themselves interpretable (e.g., a newly developed measure), as they express observed patterns in the data in a way interpretable without reference to the units of measurement.

Standardised effect size measures are also useful in power analysis and meta-analysis. In meta-analysis, the standardised effect sizes are typically the main unit of analysis and allow for a set of studies to be collapsed and assessed together. This chapter focuses on standardised effect sizes because these are typically required for power analysis. In order to perform formal sample size determination, researchers must specify an alternative hypothesis in sufficient detail to determine the sampling distribution of the test statistic under the alternative hypothesis. For relatively simple designs (e.g., for a comparison of the mean scores of two independent groups or correlational analysis) the specification of a single standardised effect size characterises the sampling distribution under the alternative hypothesis adequately for power analysis {Cohen, 1988 #562}. For more complex designs (e.g., when covariates are to be included or when repeated measures designs are used) additional parameters may need to be specified. One of the major difficulties often cited by researchers in performing a power analysis is that they have trouble developing appropriate parameters for use in power analysis [cite interviews and survey]. This chapter outlines the three main types of effect sizes, outlines some commonly maligned benchmarks that have been proposed for power analysis, and provides a systematic review of previous efforts to estimate the average effect size seen in various areas of psychology research.

Most standardized effect sizes can be grouped into three main categories. There are effect sizes for group differences (e.g., Cohen’s *d* and Hedge’s *g*), variance explained effect sizes (e.g., r, R2, eta2, partial eta2, omega2), and there are probability effect sizes (e.g., odds ratios).

**Effect sizes for Mean differences**

Cohen’s *d*, was originally proposed as an measure of the size of effect in Cohen’s first power survey, and was explicitly developed to aide in sample size determination (Cohen, 1962). There are a number of estimators for the population parameter $\delta$, the difference between groups divided by the pooled standard deviation. The estimates produced by all of these estimators are commonly called “Cohen’s *d*”, and all use equation x.1.

C:\Users\fsingletonthorn\Downloads\CodeCogsEqn (1).gif x.1

$d = \frac{\bar{x}\_1 - \bar{x}\_2}{s\_p}$

(adapted from McGrath & Meyer, 2006, p. 386)

Where $\bar{x}\_1$ is the mean of sample 1, and $\bar{x}\_2$ is the mean of sample 2, and $s$ is the pooled standard deviation. The pooled standard deviation is most often calculated for samples as:

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$$s\_p = \sqrt{\frac{\sum(x\_1 - \bar{x}\_1)^2 + (x\_2 - \bar{x}\_2)^2}{n\_1 + n\_2 - 2}}$$

(Cohen, 1977, p. 67)

Or equivalently as equation x.3[[1]](#footnote-1).

(x.3)

$$s = \sqrt{frac{(n\_1-1)s\_1^2 + (n\_2-1)s\_2^2}{n\_1 + n\_2 - 2}}$$

(adapted from Hedges, 1981, p. 110)

Where $s\_j^2$ is the sample variance for each group, calculated as

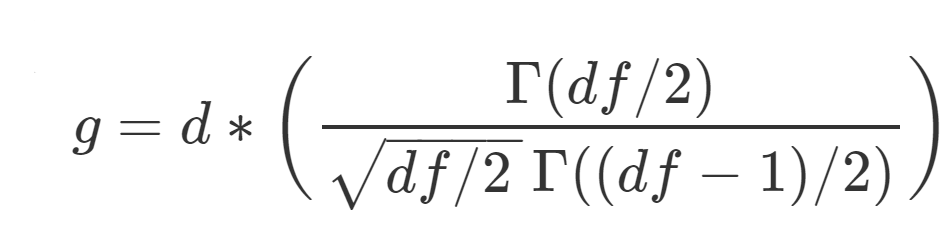
https://latex.codecogs.com/gif.latex?s%5E2_j%20%3D%20%5Cfrac%7B1%7D%7Bn_j-1%7D%20%5Cdisplaystyle%5Csum_%7Bi%3D1%7D%5E%7Bn%7D%20%28x_%7Bj%2Ci%7D%20-%20%5Cbar%7Bx%7D_j%29%5E2 (x.4)

$$s^2\_j\ =\ \frac{1}{n\_j-1}\ \displaystyle\sum\_{i=1}^{n}\ (x\_{j,i}\ -\ \bar{x}\_j)^2$$

The j subscript indicating the group.

The pooled standard deviation should be calculated for populations (i.e., if all possible units of analysis have been collected) using n1+n2 in the denominator as opposed to n1+n2-2, without Bessel’s correction (Cohen, 1977, 1988; McGrath & Meyer, 2006).

Terminology around these effect sizes is remarkably inconsistent, and sometimes Cohen’s d is reserved to describe the estimator that doesn’t use Bessel’s correction, and the estimator outlined in equation x.1 to x.3 is called Hedges’ g (e.g., (Rosenthal, 1991)). However, as Cohen outlined both estimators (e.g., Cohen, 1977) before Hedges (1981), and as the population version is rarely applicable, it seems reasonable to use Cohen’s *d* to refer to the estimator outlined in equations x.1 to x.4. This estimator for Cohen’s d is consistent (that is, as the n increases its expectation increasingly accurately approximates the population parameter), but it is upwardly biased (it tends to overestimate the population parameter, especially when the included sample size is small). Hedges (1981) outlines a correction factor to produce an unbiased estimator:

(x.5)

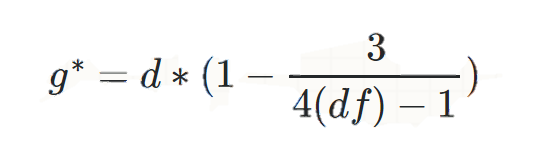
$$g = d \* (\frac{\Gamma(df/2)}{\sqrt{df/2 \,}\,\Gamma((df-1)/2)})$$

Where $df=n1+n2−2$ for an independent groups design, d is calculated as per equation x.1 and $\Gamma(x)$ is the gamma function.

(Originally from Hedges, 1981; this version adapted from Hedges & Olkin, 1985, p. 104).

However, this correction factor is fairly computationally complex (although trivial on modern computers), so Hedges also provided a computationally simple approximation which performs well for all practical scenarios (Hedges, 1981, p. 114).

Hedge’s approximate bias corrected *g*\* is calculated as:

 (x.6)

$$g^\* = d\*(1 - \frac{3}{4(df)-1})$$

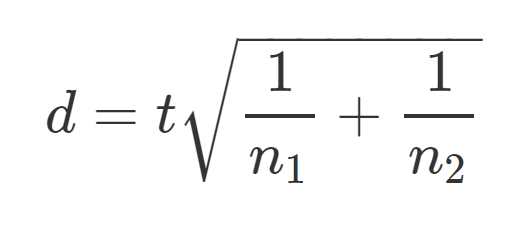
Where $df=n1+n2−2$ for an independent groups design and *d* is Cohen’s d as calculated in equation x.1 using x.2 as the estimator for the variance.

(this version adapted from Borenstein, Hedges, Higgins, & Rothstein, 2011, p. 27; originally from Hedges, 1981)

People commonly refer to $d$, $g$ and $g^\*$ as Hedge's g or Cohen's d interchangeably (Lakens, 2013). They are all virtually identical for most practical purposes when *n* > 30, and all can be interpreted in the same way. For the purposes of power analysis, it is important to realise that Cohen’s d is upwardly biased if estimating a $\Delta$ based on a literature that uses the biased estimator. Practically, for the purposes of power analysis, sampling variability and selective reporting are likely to create greater difficulties than the estimator that has been used.

**Summary statistics conversion for two group scenarios**

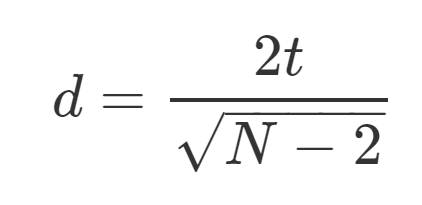
If effect sizes have not been reported, Cohen’s d can be calculated using the results of an independent samples t tests using the formula



$$d = t\sqrt{\frac{1}{n\_1}+\frac{1}{n\_2}} $$

(Lakens, 2013, equation 2)

Where $n\_1$ and $n\_2$ are the sample sizes for groups 1 and two respectively, and *t* is the result of an independent samples t test.



$$d=\frac{2t}{\sqrt{N - 2}}$$

(Rosenthal, 1991, p. 17)

Which is correct if the groups are equal, and will be an underestimate if the groups are unequal. However, although even if the ratio of samples sizes in each group is as extreme as 70 to 30 the underestimation will be no more than 8% (Rosenthal, 1991).

**Standardised mean differences for the comparisons of two repeated measures:**

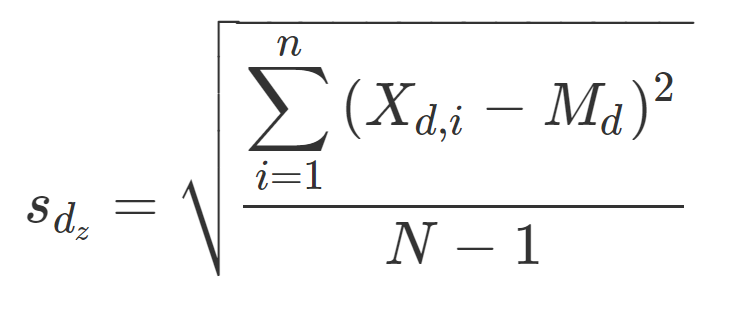
The most common effect size measure for mean difference between repeated measures is also commonly called Cohen’s d, and following Cohen (1977, 1988) I will refer to the repeated measures version as Cohen’s $d\_z$. This effect size follows a similar general form to the independent samples Cohen’s d (x.1), except the denominator is the mean difference between measures,

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$$d\_z = \frac{M\_d}{s\_d}$$

(Lakens, 2013) equation 6

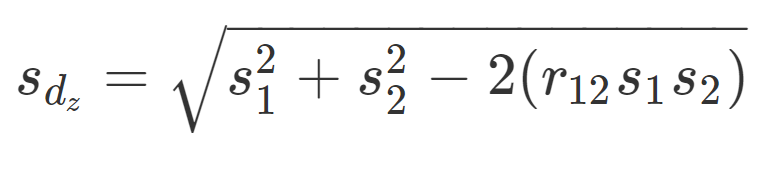
Where $M\_d$ is the mean difference score, and $s\_d$ is the standard deviation of the difference scores calculated as:

 (x.8)

$s\_{d\_z} =\sqrt{\frac{\displaystyle\sum\_{i=1}^{n}{(X\_{d,i} - M\_d)^2}}{N-1}}$

Where $\X\_{d,i}$ is the difference scores for case i, $M\_d$ is the mean difference score, and $ s\_{d\_z}$ is the standard deviation of the difference scores.

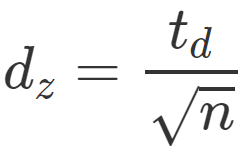
Equivalently, $s\_{d\_z}$ can be calculated as :

 (x.9)

$s\_{d\_z} = \sqrt{s\_1^2 + s\_2^2 - 2(r\_{12}s\_1s\_2)}$

(Cohen 1988 p. 48)

Where $s\_1$ and $s\_2$ are the variances of groups one and two, and $r\_{12}$ is equal to the Pearson correlation between subjects measures on measure one and measure two. Notably, this equation x.9 highlights an important fact Cohen’s $d\_z$, and repeated measures t tests, that the effect size is dependent upon the correlation between scores on repeated measures. The higher the correlation, the greater the $dz$. This can make a large difference to the $d\_z$, for example, taking the example the two groups have equal variance, and there is a one standard deviation difference between groups, $d\_z$ can vary between .707 (when $r\_{12} = 0$) and infinity (when $r\_{12}$ approaches 1). For this reason it has been argued that classical Cohen’s d (equation x.1) should be interpreted in lieu of $d\_z$ for maximum interpretability and comparability across experimental designs {Morris, 2002 #808}. However, for the purposes of power analysis, it is beneficial to use $d\_z$, due to the fact that as the correlation between repeated measures increases the standard error of the difference decreases, or equivalently, the size of the test statistic increases, as can be seen in equation [Rosen]:

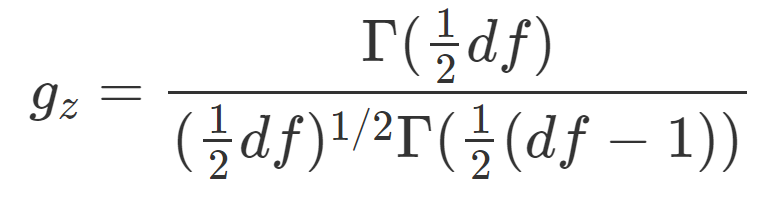
 equation [Rosen]

$$d\_z = {frac{t\_d}{sqrt{n}}} $$

(Lakens, 2013) Equation 7

Where $*t*\_d$ is the *t* statistic calculated as per a repeated measures *t* test, and *n* is the sample size.

$d\_z$ is also biased, and an equivalent to Hedges’ correction can be applied to adjust for this bias similarly to the independents samples Cohen’s d (Gibbons, Hedeker, & Davis, 1993):

 (x.Gibbons)

(equation 7, p. 274 Gibbons, Hedeker & Davis, 1993)   
$$g\_z = \frac{\Gamma(\frac{1}{2}df)}{(\frac{1}{2}df)^{1/2} \Gamma(\frac{1}{2}(df-1))}$$

Where $df =$ degrees of freedom (i.e., $n - 1$ as per repeated measures *t*-tests) and $\Gamma (x)$ is the gamma function.

Proportion overlap (U) -



*Figure [Cohen’s d as population distributions]*. Population distributions with a mean difference of .2, .5, .8 and 1.2 Cohen’s d, along with the percentage overlap between populations (calculated assuming that populations are normally distributed, have equal variance, and equal sample sizes, using equations from (Reiser & Faraggi, 1999)).

**Association/variance explained:**

**Categorical effect sizes:**

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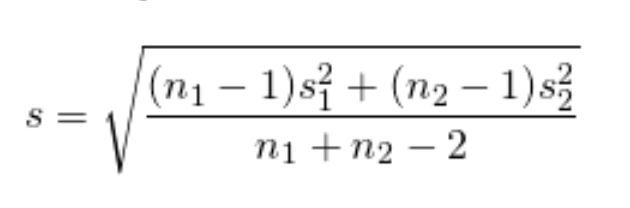
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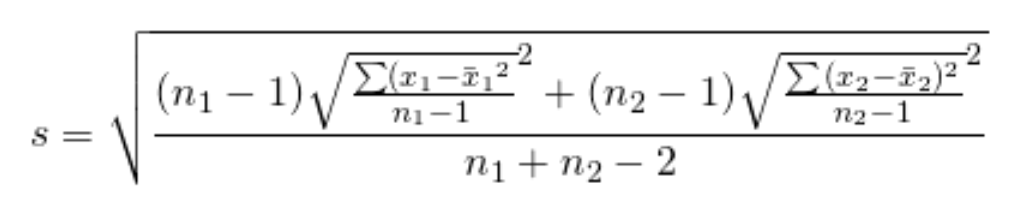
Supplementary material [Cohen’s d/Hedges’ g]

Although the equivalence between formulas x.3 and x.2 is relatively trivial, it seems worth highlighting this equivalence more explicitly as this appears to be a common source of confusion for students and researchers. For example, (Maher et al., 2013) reported that the difference between d and g is that Hedge’s g uses equation x.3 to calculate the pooled standard deviation instead of equation x.2, despite the fact that those formulas are mathematically identical.

 equation x.2

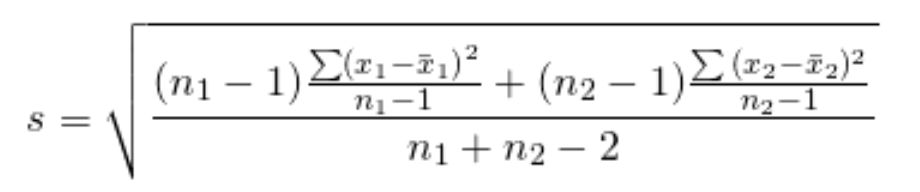
$$s = \sqrt{\frac{(n\_1 -1)s\_1^2 + (n\_2 -1)s\_2^2}{n\_1 + n\_2 - 2}} $$

However, this simplifies to equation x.2 Both “na – 1” and “nb – 1” in the numerator of the fraction cancel out, as can be seen more clearly when the s is replaced with the formula for calculating the standard deviation in [x.2 expanded].

 [x.2 expanded]

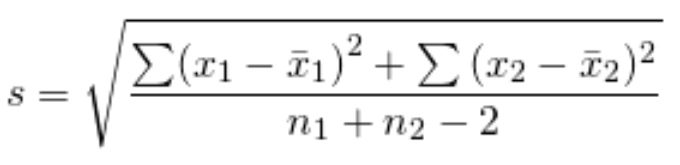
$$s = \sqrt{\frac{(n\_1 -1)\sqrt{\frac{\sum ({x\_1-\bar{x}\_1}^2}{n\_1-1}}^2 + (n\_2 -1)\sqrt{\frac{\sum {(x\_2-\bar{x}\_2)^2}}{n\_2-1}}^2}{n\_1 + n\_2 - 2}}$$

Simple algebra then simplifies this formula to equation [Simplified1].

 [simplified1]

$$ s = \sqrt{\frac{(n\_1 -1)\frac{\sum ({x\_1-\bar{x}\_1)}^2}{n\_1-1} + (n\_2 -1){\frac{\sum {(x\_2-\bar{x}\_2)^2}}{n\_2-1}}}{n\_1 + n\_2 - 2}}$$

Multiplying the elements in the numerator out, we get equation [simplified2], which is identical to equation x.2 above.

 [simplified2]

$$s = \sqrt{\frac{\sum ({x\_1-\bar{x}\_1)}^2 + {\sum {(x\_2-\bar{x}\_2)^2}}}{n\_1 + n\_2 - 2}}$$

EVERYTHING TOGETHER FOR COHEN’S D:

$$d = \frac{\bar x\_1 -\bar x\_2}{

\sqrt{\frac{\sum(x\_1 - \bar{x}\_1)^2 + (x\_2 - \bar{x}\_2)^2}{n\_1 + n\_2 - 2}}}$$

Stats exchange answer:

It seems when people say Cohen's d they mostly mean:

$$d = \frac{\bar{x}\_1 - \bar{x}\_2}{s}$$

Where $s$ is the pooled standard deviation,

$$s = \sqrt{\frac{\sum(x\_1 - \bar{x}\_1)^2 + (x\_2 - \bar{x}\_2)^2}{n\_1 + n\_2 - 2}}$$

There are other estimators for the pooled standard deviation, probably the most common apart from the above being:

$$s^\* = \sqrt{\frac{\sum(x\_1 - \bar{x}\_1)^2 + (x\_2 - \bar{x}\_2)^2}{n\_1 + n\_2}}$$

Notation here is remarkably inconsistent, but sometimes people say that the the $s^\*$ (i.e., the $n\_1 + n\_2$ version) version is called Cohen's $d$, and reserve the name Hedge's $g$ for the version that uses $s$ (i.e., with Bessel’s correction, the n1+n2−2 version). This is a bit weird as Cohen outlined both estimators for the pooled standard deviation (e.g., $s$ version on p. 67, Cohen, 1977) before Hedges wrote about them (Hedges, 1981).

Other times Hedge's g is reserved to refer to either of the bias corrected versions of a standardised mean difference that Hedges developed. Hedges (1981) showed that Cohen's d was upwardly biased (i.e., its expected value is higher than the true population parameter value), especially in small samples, and proposed a correction factor to correct for Cohen's d's bias:

Hedges's g (the unbiased estimator):

$$g = d \* (\frac{\Gamma(df/2)}{\sqrt{df/2 \,}\,\Gamma((df-1)/2)})$$

Where $df = n\_1 + n\_2 -2$ for an independent groups design, and $\Gamma$ is the gamma function.

(originally Hedges 1981, this version developed from Hedges and Olkin 1985, p. 104)

However, this correction factor is fairly computationally complex, so Hedges also provided a computationally trivial approximation that, while still biased, is only biased to an extremely small extent:

Hedges' $g^\*$ (the computationally trivial approximation):

$$ g^\* = d\*(1 - \frac{3}{4(df) - 1})$$

Where $df = n\_1 + n\_2 -2$ for an independent groups design.

(Originally from Hedges, 1981, this version from Borenstein, Hedges, Higgins, & Rothstein, 2011, p. 27)

But, as for what people mean when they say Cohen's d vs. Hedges' g vs. g\*,

people seem to refer to any of these three estimators as Hedge's g or Cohen's d interchangeably, although I've never seen someone write "$g^\*$" in a non-methodology/stats research paper. If someone says "unbiased Cohen's d", you're just going to have to take your best guess at either of the last two (and I think there might even be another approximation that has been used for Hedge's $g^\*$ too!).

They are all virtually identical if $n > 20$ or so, and all can be interpreted in the same way. For all practical purposes, unless you're dealing with really small sample sizes, it probably doesn't matter which you use (although if you can pick, you may as well use the one that I've called Hedges' g, as it is unbiased).

1. See supplementary material [Cohen’s d/Hedges’ g] for a demonstration of the equivalence between x.2 and x.3. This is explicitly provided in the supplementary material as this appears to be a common point of confusion among students and researchers (e.g., (Maher, Markey, & Ebert-May, 2013) misidentifies equation x.3 as the equation for Hedge’s *g* and contrasts that with equation x.2 of Cohen’s *d*, despite their equality.) [↑](#footnote-ref-1)